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IDENTIFICATION OF ISOMORPHISM OF KINEMATIC CHAINS USING DIFFERENT KINEMATIC PAIRS

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ABSTRACT

Mechanical engineering design involves intuition, experience, process analysis syntheses. In this paper, an attempt has been made to provide to identification of isomorphism among the kinematic chains using different kinematic pairs, by considering the examples of pump mechanisms for a manual hand pump and nosewheel assembly for a small aircraft. By using link-link form of the modify adjacency matrix and elements of the matrix was chosen as one and zero depending on the absence or presence of a connection in between the joints. The proposed method is applied for determining the characteristic polynomial equations of these mechanisms and comparing manual hand pump to the nose wheel assembly for detecting the isomorphism.

KEYWORDS: *Isomorphism, Kinematic Chain, Kinematic Pair, Modified Adjacency Matrix.*

INTRODUCTION

Recently people have done several attempts to develop different method to find out isomorphism among two kinematic chains. In a design of mechanism problem, the systematic steps are type synthesis, structural or number synthesis, and dimensional synthesis. The structural synthesis of the kinematic chains and mechanisms have been the subject of number of studies in recent years. One important aspect of structural synthesis is to develop all possible arrangements of kinematic chains and also their derived mechanisms for a given number of links and joints, so that the designers have the freedom to choose the best and optimal mechanism according to their requirements The methods proposed so far are based on adjacency matrix [1] distance matrix [2] to determine the structurally distinct mechanisms of a kinematic chain; the flow matrix method [3], and the row sum of extended adjacency matrix methods [4, 5] are also proposed. Minimum code [6], characteristic polynomial of matrix [7], identification code [8], link path code [9], path matrices [10], multivalued neural network approach [11], mixed isomorphism approach [12], hamming value [13], artificial neural network approach [14], theory of finite symmetry groups [15, 16], the representation set of links by Vijayananda [17], Interactive weighted distance approach [18], are used to characterize the kinematic chains. Recently Lu et al. [19, 20] derived valid contracted graphs with the help of characteristic strings and identified the isomorphism.

To identify the isomorphism among the kinematic chains and having different kinematic pairs and using graph theory [21-24]. The most of these methods either have lack of uniqueness or very time consuming. The flow matrix method [3] is a lengthy process to identify the distinct mechanism (DM), as 'n' flow matrices are required to be developed. The proposed method is simple and well suited for Hand computation. An additional advantages of this method is that the Bocher's formula, when interpreted on the basis of well known results of kinematic chains, reveals the physical meaning of the co-efficients of characteristic polynomial equations and thus lead to a possible way of arriving at these co-efficients by inspection of the chain itself. It uses the characteristic polynomial equations for structural analysis and the identification of isomorphism between the kinematic chains and mechanisms. This method is successfully applied to four link single degree of freedom planar kinematic chains.

In the proposed method, characteristic polynomial equations are used for equality of the links if the characteristic equations of two kinematic chains are same such that the chains are isomorphic in nature. If the equations are not same then the kinematic chains are not isomorphic.

PROPOSED METHODOLOGY
The (0, 1) adjacency matrix and the distance matrix are generally used to represent the kinematic graph of a kinematic chain. The adjacency matrix shows the only the connectivity between the adjacency vertices or links. A generalized matrix representation is the elements of adjacency matrix a_{ij} represent the types of the joints. The value of a_{ij} is 1 if the joint between the ith and j^{th} link is a simple joint, 2 if it is double joint, 3 if it is ternary joint, 4 if it is quaternary and so on.

MODOFIED ADJACENCY MATRIX
The modified adjacency matrix may be defined as,

$$[MA] \quad = \{a_{ij}\}$$

Where, a_{ij} is the joint value of the edge or vertices i^{th} vertex and j^{th} those are directly connected.
Otherwise $a_{ij} = 0$;
Also, $a_{ii} = 0$
Designation of Kinematic Pairs:
Lower pair : 3.0
Higher pair : 4.0
Further classification
For Lower Pairs:
Turning pair   : 3.1;
Sliding pair   : 2.2;
Helical pair   : 3.3;
For Higher Pairs:
Point Contact   : 4.1;
Line Contact   : 2.2;

MATRIX REPRESENTATION OF KINEMATIC CHAINS

$$A_m = \begin{bmatrix} 0 & a_{12} & a_{13} & \dots & \dots & a_{1n} \\ a_{21} & 0 & a_{23} & \dots & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & \dots & 0 \end{bmatrix}$$

Example: 1. Pump mechanism for a manual water pump:
The pump in Figure 1 is activated manually by pushing on the handle (link 3). In general, the pivoted link connected to the frame is called the *crank*. This link is not always capable of completing a full revolution. The link that translates is called the *slider*. This link is the piston/rod of the pump in Figure 1, however, is connected by three pin joints and one sliding joint. A mechanism that drives a manual water pump is shown in Figure (1a). The corresponding kinematic diagram is given in Figure (1b). Pump mechanism for a manual water pump (the slider-crank mechanism has one degree of freedom)
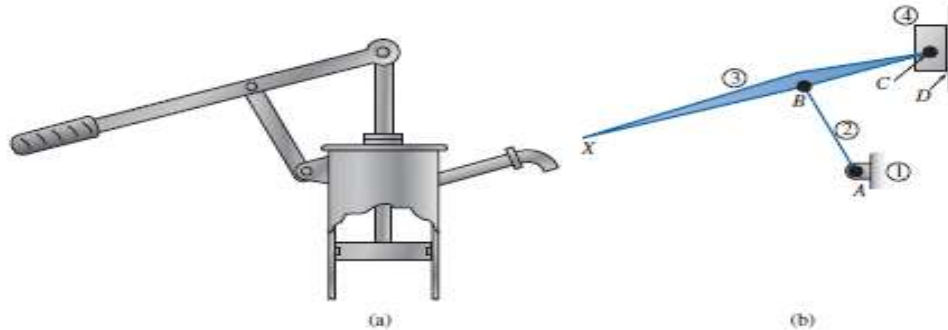
Fig 1. Pump mechanism for a manual water pump (a) Mechanism, (b) Kinematic diagram

Example:1. The matrix for manual water pump:
Substitutes the values of different kinematic pairs in the modified adjacency matrix, we obtain KP for shear press.

$$A_1 = \begin{bmatrix} 0 & 3.1 & 0 & 3.2 \\ 3.1 & 0 & 3.1 & 0 \\ 0 & 3.1 & 0 & 3.1 \\ 3.2 & 0 & 3.1 & 0 \end{bmatrix}$$

Computation for the characteristic polynomialequations:
Let the characteristic polynomial of the matrix A_1 be represented by

$a_0x^n + a_1x^{n-1} + a_2x^{n-2}\ldots\ldots\ldots\ldots + a_{n-1}x + a_n\ldots\ldots\ldots\ldots\ldots\ldots\ldots$(i)

where $a_0, a_1, a_2,\ldots\ldots a_n$ are the co-efficients of characteristic polynomial equation
Bocher's formula given by

$a_0 = 1$

$a_j = -\frac{1}{j}\sum_{r=1}^{j} a_{j-r}(S_r)_j\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$(ii)

where j=1, 2, 3, 4,……..n and $S_r = T_r(A_1{}^r)$ = Trace of matrix (A_1).
Thus to calculate the co-efficient of characteristic polynomial we need to compute the power of the matrix A   upto A^n and then use the Boacher's formula**.** The powers of matrix $[A_1]$ are $A_1{}^2$, $A_1{}^3$, $A_1{}^4$ from these powers of the matrix the traces are obtained as**,**

$S_1 = \quad 0 \qquad\qquad = \quad T_r(A_1);$
$S_2 = \quad 78.14 \qquad = \quad T_r(A_1{}^2);$
$S_3 = \quad 0 \qquad\qquad = \quad T_r(A_1{}^3);$
$S_4 = \quad 3052.5454 = \quad T_r(A_1{}^4)$

Now using the Bocher's formula
$a_0 = 1;$
$a_1 = -a_0s_1 = 0;$
$a_2 = \quad -1/2(a_1 s_1 + a_0s_2) = \quad -39.07;$
$a_3 = -1/3(a_2 s_1 + a_1 s_2 + a_0s_3) = \quad 0 ;$
$a_4 = -1/4(a_3s_1 + a_2 s_2 + a_1 s_3 + a_0s_4) = \quad 0.0961;$

so, the characteristic polynomial for the shear press is given as
$f(x) = x^4 - 39.07 + 0.0961$ ………………………(3)

Example:2. Matrix for the nosewheel assembly for a small aircraft:
In an analysis that focuses on the landing gear, the motion of the wheel assembly would be determined relative
to the body of the aircraft. Therefore, the aircraft body will be designated as the frame. Figure(2b) shows the
kinematic diagram for the nosewheel assembly for a small aircraft, numbering and labeling the links. The tip of the
wheel was designated as point of interest *X.*The degree of freedom for nosewheel assembly for a small aircraft is one.
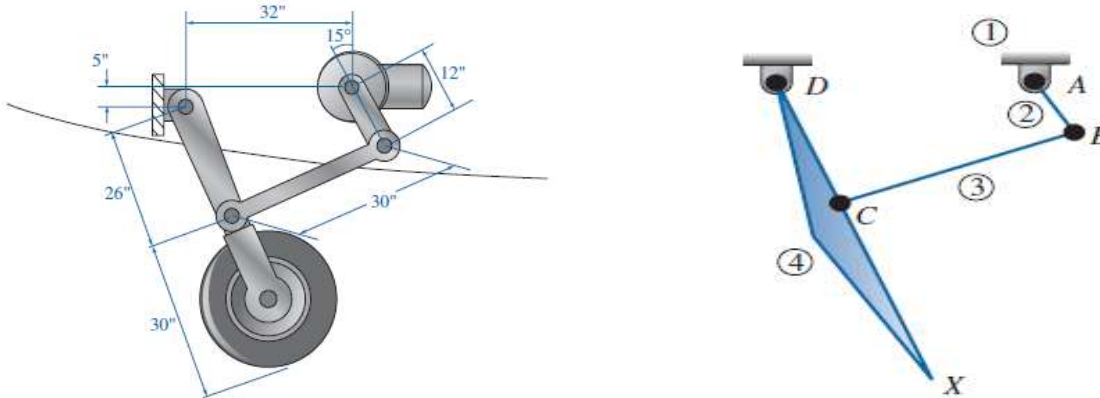
Fig. 2. (a) Nosewheel Assembly for a small aircraft (b) Kinematic Diagram.

$$A_2 = \begin{bmatrix} 0 & 3.1 & 0 & 3.1 \\ 3.1 & 0 & 3.1 & 0 \\ 0 & 3.1 & 0 & 3.1 \\ 3.1 & 0 & 3.1 & 0 \end{bmatrix}$$

The powers of matrix $[A_2]$ are A_2^2, A_2^3, A_2^4 from these powers of the matrix the traces are obtained as,

$S_1 = 0 = T_r(A_2)$;
$S_2 = 76.28 = T_r(A_2^2)$;
$S_3 = 0 = T_r(A_2^3)$;
$S_4 = 2955.2672 = T_r(A_2^4)$;

Now using the Bocher's formula

$a_0 = 1$;
$a_1 = a_0 s_{1 =} 0$;
$a_2 = -1/2(a_1\ s_1 + a_0 s_2)\ = -38.44$;
$a_3 = -1/3(a_2\ s_1 + a_1\ s_2 + a_0 s_3)\ = 0$;
$a_4 = -1/4(a_3 s_1 + a_2\ s_2 + a_1\ s_3 + a_0 s_4) = 0$;

Substituting these values in the characteristic polynomial equation so, we get the characteristic polynomial for the nosewheel assembly is given as,

$f(x) = x^4 - 38.44x^2$ ………………………(4)

Thus, from the equations (3) and (4), it is clearly shows that the manual water pump and nose wheel assembly for a small aircraft is not isomorphic having different kinematic pairs, because their characteristic polynomial equations are different.

RESULTS AND DISCUSSION
The characteristic polynomial for the shear press is given as
$f(x) = x^4 - 39.07 + 0.0961$
the characteristic polynomial for the nosewheel assembly for a small aircraft is given as,
$f(x) = x^4 - 38.44x^2$

Isomorphism:
Two chains are isomorphic only if their characteristic polynomial equations are same.
Kinematic Chain for shear press is not isomorphic with nosewheel assembly for a small aircraft chain as the characteristic equations derived are not similar.

Degree of Freedom:
The number of independent coordinates required to completely specify the relative movement.
Shear press and nosewheel assembly for a small aircraft mechanism both have degree of freedom one but does not have fractionated and partial degree of freedom chain.

CONCLUSION

We observed that the determinants of two kinematic chains or mechanisms as shown in fig 1 and 2, are different, it reveals that their characteristic polynomial equations are different for both the mechanisms. Hence these mechanisms are treated as non-isomorphic or distinct mechanisms.

This method is able to clearly identify the isomorphism among the kinematic chains or mechanisms having different kinematic pairs.

Characteristic polynomial equations have been used for identification and detection of isomorphism between two or more kinematic chains.

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